

QCD corrections to $e^+e^- \rightarrow J/\psi(\psi(2S)) + \chi_{cJ}$ ($J=0,1,2$) at B factories

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We analytically calculate the cross sections of double charmonium production in $e^+e^- \rightarrow J/\psi(\psi(2S))\chi_{cJ}$ ($J=0,1,2$) at next-to-leading order (NLO) in α_s in nonrelativistic QCD, and confirm factorization of these processes. In contrast to χ_{c0} production, for which the NLO correction is large and positive, the NLO corrections for $\chi_{c1,2}$ production can be negative, resulting in decreased K factors of 0.91 and 0.78 for $J=1$ and 2 respectively when $\mu = 2m_c$. Consequently, the NLO QCD corrections markedly enlarge the difference between cross sections of χ_{c0} and $\chi_{c1,2}$. This may explain why $e^+e^- \rightarrow J/\psi(\psi(2S))\chi_{c0}$ but not $e^+e^- \rightarrow J/\psi(\psi(2S))\chi_{c1,2}$ is observed experimentally. Moreover, for $J/\psi(\psi(2S))\chi_{c1,2}$, the NLO QCD corrections substantially reduce the μ dependence and lead to predictions with small theoretical uncertainties.

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I. INTRODUCTION

The study of heavy quarkonium inclusive production is important to understanding the hadronization of heavy quarkonium. In the spirit of nonrelativistic QCD (NRQCD) factorization [1], it is believed that both color-singlet and color-octet channels can contribute to heavy quarkonium inclusive production. However, due to the lack of knowledge of color-octet long-distance matrix elements, the inclusive production mechanism for heavy quarkonium is still not fully understood even though complete next-to-leading order (NLO) QCD corrections for most of these processes are available [2–7]. On the other hand, in heavy quarkonium exclusive production all final state particles are targeted, and in some processes the color-octet does not contribute at all, such as in double charmonium production in e^+e^- annihilation. As a result, the study of double charmonium exclusive production, among other kinds of production, may provide a good opportunity to learn more about color-singlet mechanisms for production and hadronization, without theoretical uncertainties from color-octet contributions.

In 2002, the Belle Collaboration reported a surprisingly large double charmonium production cross section[8], which was later confirmed by larger data samples[9, 10] and also by BaBar[11]. The observed double charmonium production cross sections for $e^+e^- \rightarrow J/\psi\eta_c(\chi_{c0})$ were much larger than the leading order (LO) calculations[12] in NRQCD. Aside from attempted explanations by other models and methods[13], in the framework of NRQCD these discrepancies were found to be essentially resolved by large NLO QCD corrections[14, 15] and relativistic corrections[16]. It is puzzling that, although the cross section of $J/\psi\chi_{c0}$ is observed to be very large, $J/\psi\chi_{c1,2}$ is not seen. After all, at LO the cross section of $J/\psi\chi_{c0}$ is larger than the total cross sections of $J/\psi\chi_{c1,2}$ by only a factor of a little more than 2. Even with the most recent data[17], signals of $J/\psi\chi_{c1,2}$ are still not clearly seen. Recall that there was a similar situation for double J/ψ production. At LO, NRQCD predicts the $J/\psi J/\psi$

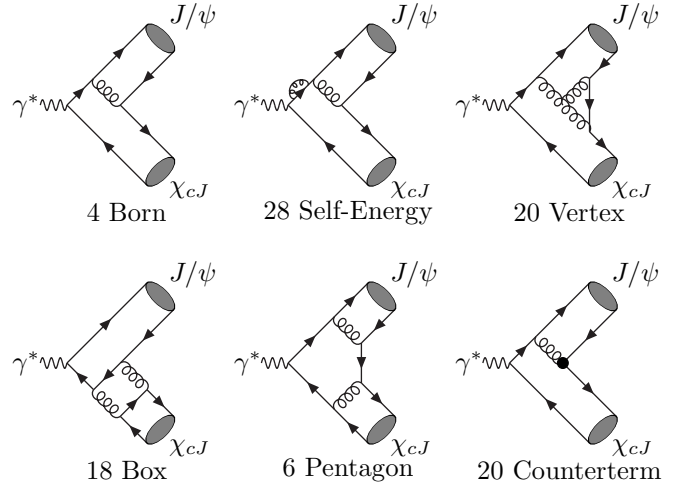


FIG. 1: Some representative Feynman diagrams for $e^+e^- \rightarrow J/\psi\chi_{cJ}$.

cross section to be larger than that of $J/\psi\eta_c$ by a factor of 1.8[18], but experimentally no evidence for double J/ψ production was observed. This puzzle was explained later by finding a negative NLO QCD correction for double J/ψ [19] and a positive NLO QCD correction for $J/\psi\eta_c$ [14, 15]. All these show that the NLO QCD corrections can be very important for double charmonium production. Therefore, it is necessary to examine the NLO QCD corrections for $J/\psi\chi_{c1,2}$ as well as $J/\psi\chi_{c0}$ production in e^+e^- annihilation and to see whether the NLO QCD corrections play important roles in understanding the nonobservation of $J/\psi\chi_{c1,2}$ production at B factories.

The NLO QCD correction for $J/\psi\chi_{c0}$ has been calculated in [15], and the difference between χ_{c0} and $\chi_{c1,2}$ in the calculation is the summation over polarizations. However, the NLO QCD correction for $J/\psi\chi_{c1,2}$ is indeed a nontrivial work even though that for $J/\psi\chi_{c0}$ has been

achieved. This is because χ_{c0} is a scalar particle, so the summation over its spin and angular polarizations can be done at the amplitude level, which makes its Lorentz structure much simpler than that of $\chi_{c1,2}$. However, the method used to calculate $J/\psi\chi_{c0}$ in [15] can hardly be adopted here to calculate $J/\psi\chi_{c1,2}$. Furthermore, the method in [15] gives only numerical results, while an analytical expression is really important for analyzing the details of the result. Fortunately, using our recently developed method, which has been used to do the complete NLO corrections for heavy quarkonium hadroproduction [5], analytical expressions of the production cross sections of $e^+e^- \rightarrow J/\psi(\psi(2S))\chi_{cJ}$ at NLO can be conveniently achieved.

II. CALCULATION

In the following, we briefly describe our calculation. We use **FeynArts**[20] to generate Feynman diagrams and Feynman amplitudes. Some representative Feynman diagrams for this process are shown in Fig. 1. There are generally ultraviolet(UV), infrared(IR), and Coulomb singularities. Conventional dimensional regularization (CDR) with $D = 4 - 2\epsilon$ is adopted to regularize them. The UV-divergences from self-energy and triangle diagrams are removed by renormalization. The Coulomb singular terms are factored into the J/ψ and χ_{cJ} wave functions. Differing from S-wave charmonium production, where IR singularities can be canceled to ensure factorization, there are generally nonfactorizable IR divergences in P-wave charmonium exclusive production (such as $B \rightarrow \chi_{cJ}K$ [21]) due to the nonvanishing relative momentum between the heavy quark and the antiquark. As previously discussed in [15], it is the existence of the associated S-wave state J/ψ that avoids topologically nonfactorizable soft interactions and leads to cancellation of IR singularities. This argument is generalized in [22] to prove the factorization theorem of heavy quarkonium exclusive production to all orders in α_s . In the present work, we find that the IR singularities are indeed canceled between different diagrams, which further confirms factorization of $e^+e^- \rightarrow J/\psi\chi_{cJ}$ ($J = 0, 1, 2$).

To perform the calculation, two different methods are used, resulting in two completely independent computer codes. The results are found to agree with each other with high precision. Furthermore, our result for χ_{c0} is basically consistent with that in [15]^a.

One of our calculations is based on the **Mathematica** package **FeynCalc**[23]. We separate the soft singularities in the virtual corrections using the method used in [24], and then we treat the singular part analytically and the finite part numerically. To calculate the finite

part, we use the traditional method [25] to reduce tensor loop integrations to scalar functions, which are then calculated numerically by **LoopTools**[26]. When derivatives of finite scalar functions are needed, we perform them numerically. This method involves large numerical cancellations, so we use quadruple precision in the calculation to guarantee the result to be reliable. The other one of our calculations is based on our self-written **Mathematica** codes. The derivatives are performed before tensor reductions and loop integrations, so it avoids the derivatives of scalar functions. Then we decompose tensor integrals and reduce scalar functions to a fundamental set using integrate-by-part-based recursion relations. The fundamental set consists of only one, two, and three point scalar functions in this work. We use the analytical expressions in [27] for divergent scalar functions, while finite scalar functions are calculated analytically with the methods in [28]. We use **QCDLoop**[27] to check our analytical expressions of scalar functions. Final analytical results for the cross sections are given in the Appendix.

III. RESULT AND DISCUSSION

In the numerical calculation, we set $\sqrt{s} = 10.6$ GeV, $\Lambda_{\overline{MS}}^{(4)} = 338$ MeV, and $m_c = 1.5$ GeV. The values of wave functions squared at the origin (or their derivatives) are extracted from the leptonic width of J/ψ and $\psi(2S)$ and the two-photon width of χ_{c2} [29] at NLO in α_s :

$$\Gamma_{\psi(nS) \rightarrow e^+e^-} = \frac{4\alpha^2}{9m_c^2} \left(1 - \frac{16}{3} \frac{\alpha_s}{\pi}\right) |R_{nS}(0)|^2, \quad (1)$$

$$\Gamma_{\chi_{c2}(nP) \rightarrow \gamma\gamma} = \frac{64\alpha^2}{45m_c^4} \left(1 - \frac{16}{3} \frac{\alpha_s}{\pi}\right) |R'_{nP}(0)|^2. \quad (2)$$

Setting $\alpha_s = 0.259$ and $\alpha = \frac{1}{137}$ we get

$$|R_{1S}(0)|^2 = 0.418 \text{GeV} m_c^2, \quad (3)$$

$$|R_{2S}(0)|^2 = 0.179 \text{GeV} m_c^2, \quad (4)$$

$$|R'_{1P}(0)|^2 = 0.0116 \text{GeV} m_c^4. \quad (5)$$

Note that the values of wave functions squared at the origin here depend on charm quark mass m_c . This treatment can largely cancel the m_c dependence in the short-distance coefficients and significantly reduce the theoretical uncertainty due to the choice of m_c .

In Table I, we give the ratio of cross section at NLO to that at LO (the K factor). Different from the χ_{c0} production, where the K factor is much larger than 1, the K factors for $\chi_{c1,2}$ are small, and the NLO corrections are small and even negative when $\mu = 2m_c$. Therefore, the gap of cross sections between χ_{c0} and $\chi_{c1,2}$ is further enlarged at NLO, which gives an explanation for why only $J/\psi(\psi(2S))\chi_{c0}$ production is observed. In Fig. 2, we show the cross sections at LO and NLO as functions of

^a There is an error in the numerical calculation in [15]. Our result is consistent with it only after correcting the error in [15].

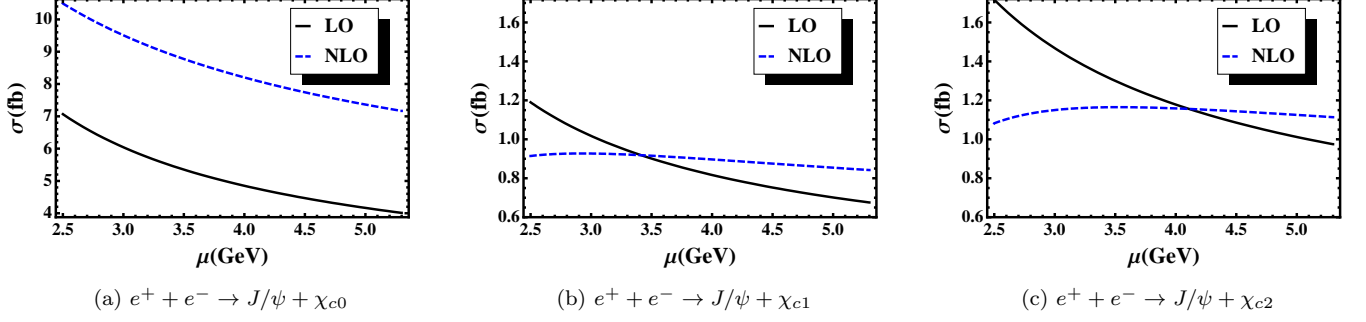


FIG. 2: Cross sections of $e^+e^- \rightarrow J/\psi\chi_{cJ}$ as functions of the renormalization scale μ at LO and NLO in α_s with $\sqrt{s} = 10.6\text{GeV}$, $\Lambda = 338\text{MeV}$, and $m_c = 1.5\text{GeV}$.

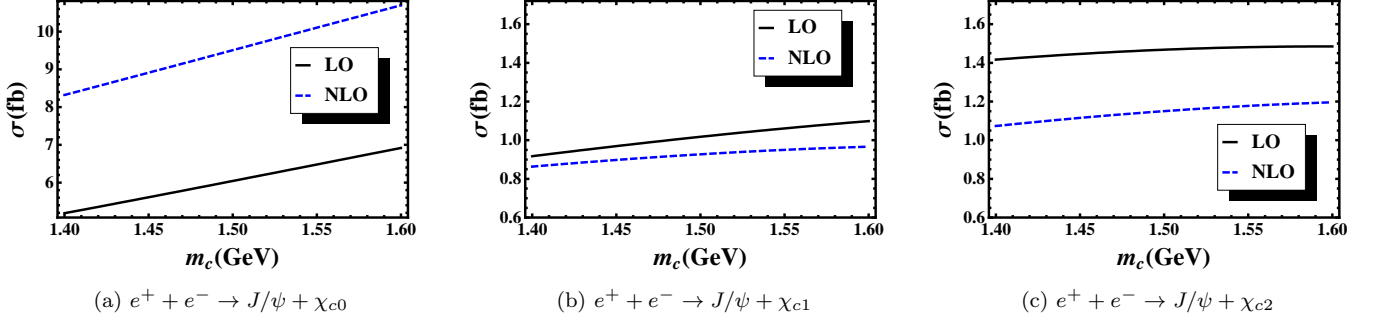


FIG. 3: Cross sections of $e^+e^- \rightarrow J/\psi\chi_{cJ}$ as functions of the charm quark mass m_c at LO and NLO in α_s with $\sqrt{s} = 10.6\text{GeV}$, $\Lambda = 338\text{MeV}$, and $\mu = 2m_c$.

the renormalization scale μ . We find that, although μ dependence for χ_{c0} is large at both LO and NLO, QCD corrections substantially reduce the μ dependence for $\chi_{c1,2}$. The dependence of charm quark mass is shown in Fig. 3, where we find the dependence for $\chi_{c1,2}$ is also weaker than that for χ_{c0} . Based on Figs. 2 and 3, we conclude that the NLO predictions for $\chi_{c1,2}$ production have small theoretical uncertainties; therefore, they can be used to precisely test the production mechanism when they can be measured in future experiment.

	$\alpha_s(\mu)$	$J/\psi + \chi_{c0}$	$J/\psi + \chi_{c1}$	$J/\psi + \chi_{c2}$
$\mu = 2m_c$	0.259	1.57	0.91	0.78
$\mu = \sqrt{s}/2$	0.211	1.79	1.25	1.14

TABLE I: The K factor of our QCD corrections, with $\sqrt{s} = 10.6\text{GeV}$, $\Lambda = 338\text{MeV}$, and $m_c = 1.5\text{GeV}$.

The comparison of experimental data with our theoretical predictions is shown in Table II, where we vary $m_c = 1.5 \pm 0.1\text{ GeV}$ to estimate the theoretical uncertainties. Our results are consistent with all data except $e^+e^- \rightarrow \psi(2S)\chi_{c0}$. The Belle Collaboration's observation that the cross section of $e^+e^- \rightarrow \psi(2S)\chi_{c0}$ is bigger than that of $e^+e^- \rightarrow J/\psi\chi_{c0}$ cannot be explained in NRQCD at LO in v , the relative velocity of charm

quark and anticharm quark, because the only difference between $\psi(2S)$ and J/ψ at LO in v is the wave functions squared at the origin, which are well estimated. Perhaps the relativistic corrections might give some hint, but the large errors in experiment should be reduced before any definite conclusion can be drawn.

IV. SUMMARY

In NRQCD, using an improved method, we get compact analytical expressions of double charmonium production cross sections of $e^+e^- \rightarrow J/\psi(\psi(2S))\chi_{cJ}$ ($J=0,1,2$) at NLO in α_s and LO in v . Moreover, we further confirm factorization of these processes. With $\sqrt{s} = 10.6\text{GeV}$, $m_c = 1.5\text{GeV}$, and $\mu = 2m_c$, we find that the cross section for χ_{c0} is enhanced by a K factor of 1.57, while the cross sections of $\chi_{c1,2}$ are decreased with K factors of 0.91 and 0.78 respectively. The large positive NLO correction to χ_{c0} and negative NLO corrections to $\chi_{c1,2}$ markedly enlarge the difference between the cross sections of χ_{c0} and $\chi_{c1,2}$ and provide an explanation for the phenomenon that only $J/\psi(\psi(2S))\chi_{c0}$ but not $J/\psi(\psi(2S))\chi_{c1,2}$ production is observed. Considering the substantially reduced μ dependence, and also the small m_c dependence, the NLO predictions for $e^+e^- \rightarrow J/\psi(\psi(2S))\chi_{c1,2}$ are more pre-

	Belle	BaBar	LO result	NLO result	LO result	NLO result
	$\sigma \times \mathcal{B}_{>2(0)}[10]$	$\sigma \times \mathcal{B}_{>2}[11]$	$(\mu = 2m_c)$	$(\mu = 2m_c)$	$(\mu = \sqrt{s}/2)$	$(\mu = \sqrt{s}/2)$
$\sigma(J/\psi + \chi_{c0})$	$6.4 \pm 1.7 \pm 1.0$	$10.3 \pm 2.5^{+1.4}_{-1.8}$	6.0 ± 0.9	9.5 ± 1.2	4.0 ± 0.8	7.2 ± 1.2
$\sigma(J/\psi + \chi_{c1})$	-	-	$1.02^{+0.08}_{-0.10}$	$0.93^{+0.04}_{-0.07}$	$0.68^{+0.09}_{-0.10}$	$0.84^{+0.07}_{-0.09}$
$\sigma(J/\psi + \chi_{c2})$	-	-	$1.47^{+0.01}_{-0.05}$	$1.15^{+0.05}_{-0.08}$	$0.97^{+0.07}_{-0.08}$	$1.11^{+0.05}_{-0.07}$
$\sigma(J/\psi + \chi_{c1}) + \sigma(J/\psi + \chi_{c2})$	< 5.3 at 90% C.L.	-	$2.49^{+0.09}_{-0.16}$	$2.08^{+0.08}_{-0.14}$	$1.65^{+0.16}_{-0.19}$	$1.96^{+0.11}_{-0.17}$
$\sigma(\psi(2S) + \chi_{c0})$	$12.5 \pm 3.8 \pm 3.1$	-	2.6 ± 0.4	4.1 ± 0.5	$1.7^{+0.4}_{-0.3}$	3.1 ± 0.5
$\sigma(\psi(2S) + \chi_{c1})$	-	-	$0.44^{+0.03}_{-0.05}$	$0.40^{+0.01}_{-0.03}$	0.29 ± 0.04	$0.36^{+0.03}_{-0.04}$
$\sigma(\psi(2S) + \chi_{c2})$	-	-	$0.63^{+0.01}_{-0.02}$	$0.49^{+0.02}_{-0.03}$	$0.42^{+0.02}_{-0.04}$	$0.48^{+0.02}_{-0.04}$
$\sigma(\psi(2S) + \chi_{c1}) + \sigma(\psi(2S) + \chi_{c2})$	< 8.6 at 90% C.L.	-	$1.06^{+0.05}_{-0.06}$	$0.89^{+0.04}_{-0.06}$	$0.71^{+0.06}_{-0.08}$	$0.84^{+0.05}_{-0.07}$

TABLE II: Comparison of our predicted cross sections with experiments at B factories in units of fb. The errors of our theoretical predictions are only from variations of the charm quark mass $m_c = 1.5 \pm 0.1 \text{ GeV}$. The experimental data are cross sections times branching fractions for χ_{cJ} decay into more than 2 charged tracks, while the Belle data of $\psi(2S) + \chi_{cJ}$ in [10] correspond to χ_{cJ} decay into at least 1 charged track.

cise than for many other processes and may be used to test the heavy quarkonium production mechanism in the future. However, the predicted cross section for $\psi(2S)\chi_{c0}$ production is much smaller than the data. In order to clarify this problem, we suggest that both more careful measurement be performed for this process and further theoretical investigations be made in the future.

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Note added in proof.—After this work was submitted for publication, a preprint [30] appeared in which the obtained result is consistent with ours.

Appendix: Analytical Result

Here we present the analytical result of our calculation. For brevity of expression, we define

$$\begin{aligned}
 a &= \frac{\sqrt{s}}{4m_c}, b = \sqrt{a^2 - 1}, c = \sqrt{4a^2 - 1}, d = 8a^2 + 1, \\
 e &= 2a^2 - 1, f = 8a^4 + 4(2a^2 + 1)ab + 1, \\
 g &= 2ab + 1, h = 8a^4 - 8a^2 - 4abe + 1.
 \end{aligned}
 \tag{A.1}$$

The LO cross sections for $e^+ + e^- \rightarrow J/\psi + \chi_{cJ}$ are

$$\sigma_{\chi_{cJ}}^{\text{LO}} = \frac{b\pi\alpha^2\alpha_s^2}{3888a^{15}m_c^{10}} |R_S(0)|^2 |R'_P(0)|^2 M_J^{\text{LO}}, \tag{A.2}$$

where M_J^{LO} for χ_{c0} , χ_{c1} , and χ_{c2} are given by

$$M_0^{\text{LO}} = 16a^8 + 728a^6 - 428a^4 + 38a^2 + 9, \tag{A.3}$$

$$M_1^{\text{LO}} = 192a^6 - 288a^4 + 78a^2 + 27, \tag{A.4}$$

$$M_2^{\text{LO}} = 32a^8 + 160a^6 - 376a^4 + 154a^2 + 45. \tag{A.5}$$

The NLO cross sections are

$$\sigma_{\chi_{cJ}}^{\text{NLO}} = \sigma_{\chi_{cJ}}^{\text{LO}} \left(1 + \frac{\alpha_s \text{Re} M_J^{\text{NLO}}}{72a^3 b d^3 e \pi M_J^{\text{LO}}} \right), \tag{A.6}$$

where M_J^{NLO} (only real parts are guaranteed to be correct) are given by

$$\begin{aligned}
M_0^{\text{NLO}} = & -24(256a^{12} + 7840a^{10} - 14284a^8 + 8340a^6 - 1943a^4 + 69a^2 + 30)a^2d^3l_1 + 96(32a^8 - 1420a^6 \\
& + 1710a^4 - 321a^2 - 45)a^4d^3el_2 - 24(64a^{10} + 2904a^8 + 636a^6 - 784a^4 + 42a^2 + 9)a^2d^3el_3 \\
& - 216(40a^4 - 4a^2 - 3)a^6d^3l_4 + 12(256a^{12} + 19328a^{10} - 25596a^8 + 10840a^6 - 1537a^4 - 45a^2 \\
& + 21)a^2d^3l_5 + 12(19328a^{10} - 33644a^8 + 17748a^6 - 2867a^4 - 111a^2 + 30)a^2d^3l_6 + 24(96a^{10} \\
& + 768a^8 - 812a^6 + 430a^4 - 66a^2 - 9)a^2d^3el_7 + 4(13824a^{12} + 273152a^{10} - 152936a^8 + 20260a^6 \\
& + 4376a^4 - 1059a^2 - 108)abd^2e + 2(192a^{10} + 11200a^8 - 13254a^6 + 3883a^4 - 81a^2 - 48)a^2d^3e\pi^2 \quad (\text{A.7}) \\
& - 8(6002688a^{12} + 1326848a^{10} - 2958336a^8 - 147576a^6 + 198772a^4 + 31524a^2 + 1377)a^3bde \ln 2 \\
& - 8(3035136a^{12} + 3154432a^{10} - 2263296a^8 - 276912a^6 + 133688a^4 + 24456a^2 + 1125)a^3bde \ln a \\
& - 16(8a^{12} + 656a^{10} - 2040a^8 + 4108a^6 - 1796a^4 + 126a^2 + 27)d^3e \ln(a+b) + 16(11752a^8 \\
& + 5628a^6 - 8364a^4 + 1430a^2 + 147)a^2bcd^3e \ln(2a+c) + 600(16a^8 + 728a^6 - 428a^4 + 38a^2 \\
& + 9)a^3bd^3e \ln(\mu/m_c),
\end{aligned}$$

$$\begin{aligned}
M_1^{\text{NLO}} = & +36(896a^8 + 160a^6 - 1200a^4 + 125a^2 + 60)a^2d^3el_1 + 144(848a^6 + 358a^4 - 471a^2 - 81)a^4d^3el_2 \\
& - 72(1648a^8 - 550a^6 - 436a^4 + 18a^2 + 9)a^2d^3el_3 + 18(704a^8 - 1976a^6 + 1270a^4 + 73a^2 \\
& - 42)a^2d^3el_5 - 18(4288a^8 + 1592a^6 - 3084a^4 - 199a^2 + 60)a^2d^3el_6 - 72(160a^8 - 100a^6 - 162a^4 \\
& + 42a^2 + 9)a^2d^3el_7 + 12(47104a^{12} - 164608a^{10} + 83232a^8 + 36524a^6 - 2732a^4 - 1599a^2 \\
& - 108)abde - 3(4928a^8 + 1192a^6 - 3732a^4 - 31a^2 + 96)a^2d^3e\pi^2 - 72(524288a^{14} - 2965504a^{12} \quad (\text{A.8}) \\
& - 1094656a^{10} + 1111040a^8 + 689584a^6 + 145600a^4 + 13334a^2 + 459)a^3be \ln 2 - 72(262144a^{14} \\
& - 311296a^{12} - 1196032a^{10} + 335616a^8 + 419712a^6 + 104776a^4 + 10404a^2 + 375)a^3be \ln a \\
& - 48(112a^{10} - 780a^8 + 1173a^6 - 544a^4 + 45a^2 + 27)d^3e \ln(a+b) + 48(384a^8 - 592a^6 - 946a^4 \\
& + 1010a^2 + 147)a^2bcd^3e \ln(2a+c) + 1800(64a^6 - 96a^4 + 26a^2 + 9)a^3bd^3e \ln(\mu/m_c),
\end{aligned}$$

$$\begin{aligned}
M_2^{\text{NLO}} = & -12(1024a^{12} + 4864a^{10} + 896a^8 + 11592a^6 - 7070a^4 - 249a^2 + 300)a^2d^3l_1 + 48(128a^8 - 1528a^6 \\
& - 4578a^4 - 1833a^2 - 369)a^4d^3el_2 - 24(128a^{10} + 600a^8 - 7794a^6 - 1862a^4 + 30a^2 + 45)a^2d^3el_3 \\
& - 648(4a^2 + 1)a^4b^2d^3l_4 + 6(1024a^{12} + 13376a^{10} - 16224a^8 + 17908a^6 - 5416a^4 - 867a^2 \\
& + 210)a^2d^3l_5 + 6(17600a^{10} + 31408a^8 + 7944a^6 - 11450a^4 - 1725a^2 + 300)a^2d^3l_6 + 24(192a^{10} \\
& + 720a^8 - 100a^6 + 230a^4 - 150a^2 - 45)a^2d^3el_7 + 4(147456a^{14} + 825344a^{12} - 1544320a^{10} \\
& + 468720a^8 + 189436a^6 - 20996a^4 - 8643a^2 - 540)abde + (768a^{10} + 11680a^8 + 19704a^6 + 15160a^4 \\
& + 633a^2 - 480)a^2d^3e\pi^2 - 8(11304960a^{14} + 25518080a^{12} + 43710976a^{10} + 31362816a^8 + 11667664a^6 \\
& + 2211824a^4 + 199902a^2 + 6885)a^3be \ln 2 - 8(5898240a^{14} + 17588224a^{12} + 12001280a^{10} + 13653888a^8 \\
& + 7211936a^6 + 1604320a^4 + 156660a^2 + 5625)a^3be \ln a + 16(1712a^{12} + 1016a^{10} - 3984a^8 + 469a^6 \\
& + 1264a^4 - 387a^2 - 135)d^3e \ln(a+b) + 16(1808a^8 + 3192a^6 + 5286a^4 + 4486a^2 \\
& + 735)a^2bcd^3e \ln(2a+c) + 600(32a^8 + 160a^6 - 376a^4 + 154a^2 + 45)a^3bd^3e \ln(\mu/m_c), \quad (\text{A.9})
\end{aligned}$$

where l_1 – l_7 are combinations of dilogarithms:

$$\begin{aligned}
l_1 &= \text{Li}_2(1 + b/a), l_2 = \text{Li}_2(1 - b/a), l_3 = \text{Li}_2(e + 2ab) - \text{Li}_2(e - 2ab), \\
l_4 &= 2\text{Li}_2(g) - \text{Li}_2(dg/f) + \text{Li}_2(1/h) - \text{Li}_2(g/h) - \text{Li}_2(h) + \text{Li}_2(dh/f), \\
l_5 &= \text{Li}_2\left[\frac{(a+b)^2}{e+bc}\right] - \text{Li}_2\left[\frac{(a-b)^2}{e+bc}\right] + \text{Li}_2\left[\frac{(a+b)^2}{e-bc}\right] - \text{Li}_2\left[\frac{(a-b)^2}{e-bc}\right], \\
l_6 &= \text{Li}_2\left(4a^2 - 3 - \frac{bc^2}{a}\right) + \text{Li}_2\left(\frac{4a^3 + 5a - bc^2}{ad}\right) - \text{Li}_2\left(\frac{d}{32a^4 - 24a^2 - 8bc^2a + 1}\right), \\
l_7 &= \text{Li}_2\left(\frac{a^2 + ab + 1}{2}\right) - \text{Li}_2\left(\frac{a^2 - ab + 1}{2}\right) + \text{Li}_2\left(\frac{a^2 + ab + 1}{2a^2}\right) - \text{Li}_2\left(\frac{a^2 - ab + 1}{2a^2}\right) \\
&\quad - 2\text{Li}_2\left(1 - \frac{b}{2a}\right) + \text{Li}_2\left(a^2 - ba - \frac{b}{2a}\right) + \text{Li}_2\left[\frac{14a^3 + 4a - (2a^2 + 1)b}{2ad}\right] - \text{Li}_2\left(\frac{f}{d}\right).
\end{aligned} \tag{A.10}$$

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